

Distributed model predictive control of leader-follower systems using an interior point method with efficient computations

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Abstract—Standard model predictive control strategies imply the online computation of control inputs at each sampling instance, which traditionally limits this type of control scheme to systems with slow dynamics. This paper focuses on distributed model predictive control for large-scale systems comprised of interacting linear subsystems, where the online computations required for the control input can be distributed amongst them. A model predictive controller based on a distributed interior point method is derived, for which every subsystem in the network can compute stabilizing control inputs using distributed computations. We introduce local terminal sets and cost functions, which together satisfy distributed invariance conditions for the whole system, that guarantees stability of the closed-loop interconnected system. We show that the synthesis of both terminal sets and terminal cost functions can be done in a distributed framework.

I. INTRODUCTION

Model predictive control (MPC) is a well established method of process control that has proven to be useful in numerous industrial applications in the past decades. One of the advantages of MPC is that it can be applied to large scale systems, with a considerable number of states and inputs for which hard constraints are often required [18], [22].

MPC requires that the control input at each time step be calculated by the online solution of an optimization problem. As a result, one of the drawbacks of MPC as a control algorithm is the delay introduced by the computation time that is imposed for the evaluation of functions, their first or second order derivatives, and for matrix operations, computations that are usually required for most optimization algorithms. This computational burden is also worsened when MPC is implemented for a large-scale plant of interconnected subsystems, case where the dimension of the MPC problem is multiplied by the number of subsystems. For certain industries for which the manufacturing process is slow in nature, this computational time is not an issue.

However, multi-system applications have arisen where computing the input rapidly is essential for efficiency and stability. Control problems for networks of interconnected multi-agent systems such as traffic control [10], building anti-earthquake systems [17], satellite formation flight [20], and wind turbine farms [19], have received plenty of interest in recent years. Due to the large number of inputs and outputs of this class of systems, distributed control is often required.

Efficient distributed optimization methods for solving such control problems can be found in [2], [11], [23], [24], [25]. From a practical viewpoint, such methods can be sped up by implementing stronger, more powerful computational hardware. Recent results in [3], [4], [7], [12], [23], [24], [27] however, have shown that by exploiting the special underlying structure of some MPC problems, the number of flops required for an algorithm can be reduced substantially, thus making MPC a more attractive solution for control problems where speed is essential. The authors in [4] propose an interior point method approach for solving the MPC problem in which they use a discrete-time Riccati recursion to solve the linear equations efficiently at each iterate. In [3] the authors propose a more efficient approach to linear algebra computations w.r.t the derivation given in [27]. Computational burden can be also overcome by distributing the necessary operations amongst different agents. To this purpose, the authors in [23] examine a distributed approach to optimal control problems and appropriate optimization methods.

In this paper, we focus on extending these recent results on the computational time required for the control action for MPC problems with a special underlying structure arising in large-scale leader-follower systems, where the computational burden is distributed amongst the comprising subsystems, thus providing a certain independence that is usually required for these subsystems. In the first part of the paper, a stability analysis for leader-follower systems is presented, based on a linear feedback law, that allows us to construct local terminal sets and cost functions in a completely distributed way. Compared with the existing approaches based on an end point constraint, we reduce the conservatism by combining the underlying structure of the system with distributed optimization. This leads to a larger region of attraction for the controller. Then, we formulate a distributed MPC problem for this type of systems, using a terminal cost-terminal set approach and an efficient implementation of an interior-point algorithm using Mehrotra's predictor-corrector scheme for solving the corresponding optimization problem is presented. In particular, we show how the underlying Newton system can be solved in a distributed manner.

The paper is organized as follows. In Section II we present the formulation of the MPC problem corresponding to systems of the leader-follower type and then we investigate the stability issue for the current system in a distributed manner via a linear feedback law using a structured Lyapunov function approach. In Section II-B we focus on decomposing the terminal state constraints required for stability as a

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Cartesian product using distributed set computations, after which we formulate the general centralized MPC problem. We then show how to restructure the original MPC problem in Section III as to provide computational benefits using a distributed version of an interior-point algorithm presented in Section IV.

II. DISTRIBUTED MPC USING THE TERMINAL-COST, TERMINAL SET APPROACH

Large scale systems have attracted much interest from the control systems community in recent decades. In this paper, we focus on large scale systems of leader-follower type. The MPC problem associated with leader-follower systems can be found in a number of current applications such as platoons of vehicles [26], which is of great interest in the development of automated highway systems [10], or in the renewable energy industry such as the problem of controlling a wind turbine farm [19].

Platoon or leader-follower systems imply that each subsystem, from the second one onwards, is influenced by the previous. We consider linear time invariant systems, for which the dynamics of the first subsystem are:

$$x_{t+1}^1 = A^1 x_t^1 + B^1 u_t^1. \quad (1)$$

The dynamics for the remaining $M - 1$ subsystems are described by the following linear equations:

$$x_{t+1}^i = A^i x_t^i + B^i u_t^i + A^{i,i-1} x_t^{i-1} + B^{i,i-1} u_t^{i-1}, \quad (2)$$

where $x_t^i \in \mathbb{R}^{n_i}$ and $u_t^i \in \mathbb{R}^{m_i}$ are the state and input vectors of subsystem i at time t , $A^i \in \mathbb{R}^{n_i \times n_i}$ and $B^i \in \mathbb{R}^{n_i \times m_i}$ are the state and input dynamic matrices for subsystem i , while $A^{i,i-1} \in \mathbb{R}^{n_i \times n_{i-1}}$ and $B^{i,i-1} \in \mathbb{R}^{n_i \times m_{i-1}}$ are the matrices for the coupling dynamics which define the influence of subsystem $i - 1$ upon subsystem i . For these systems, we consider mixed state and input constraints of the following polyhedral form:

$$G_x^i x_t^i + G_u^i u_t^i \leq b^i, \quad (3)$$

where $G_x^i \in \mathbb{R}^{q_i \times n_i}$, $G_u^i \in \mathbb{R}^{q_i \times m_i}$, the matrix $[G_x^i \ G_u^i] \in \mathbb{R}^{q_i \times n_i + m_i}$ has full row rank and $b^i > 0$. We employ stage cost functions for states and inputs of the quadratic form¹:

$$\ell^i(x_t^i, u_t^i) = \frac{1}{2} \left(\|x_t^i\|_{Q^i}^2 + \|u_t^i\|_{R^i}^2 \right),$$

where $Q^i \in \mathbb{R}^{n_i \times n_i}$ and $R^i \in \mathbb{R}^{m_i \times m_i}$ are positive definite. For the stability analysis, we also express the dynamics for the entire system as follows:

$$\mathbf{x}_{t+1} = \mathbf{A} \mathbf{x}_t + \mathbf{B} \mathbf{u}_t, \quad (4)$$

where $\mathbf{x}_t \in \mathbb{R}^n$ and $\mathbf{u}_t \in \mathbb{R}^m$ comprise the states and inputs of all the subsystems at time t and the matrices \mathbf{A} and \mathbf{B} are block banded matrices comprised of A^i , $A^{i,i-1}$ and B^i , $B^{i,i-1}$ respectively. In a similar fashion we define the block diagonal matrices \mathbf{Q}_d and \mathbf{R}_d comprised of Q^i and R^i , respectively. In order to ensure stability for the MPC

scheme that we define below, we use a terminal set-terminal cost approach [18], [22]. We define the following final stage cost of the form:

$$\ell_f(\mathbf{x}) = \|\mathbf{x}\|_{\mathbf{P}_d}^2,$$

where matrix $\mathbf{P}_d \in \mathbb{R}^{n \times n}$ is positive definite. In order to find \mathbf{P}_d and also a terminal set X_f we search for a linear feedback law $\mathbf{u}_t = \mathbf{K}_d \mathbf{x}_t$, such that the system

$$\mathbf{x}_{t+1} = (\mathbf{A} + \mathbf{B} \mathbf{K}_d) \mathbf{x}_t \quad (5)$$

satisfies the following three properties [18]:

$$\text{A.1 } \{(\mathbf{x}, \mathbf{K}_d \mathbf{x}) | \mathbf{x} \in X_f\} \subseteq \{(\mathbf{x}, \mathbf{u}) | G_x^i x^i + G_u^i u^i \leq b^i\}$$

$$\text{A.2 } (\mathbf{A} + \mathbf{B} \mathbf{K}_d) \mathbf{x} \in X_f, \forall \mathbf{x} \in X_f$$

A.3 ℓ_f satisfies the following property:

$$\ell_f((\mathbf{A} + \mathbf{B} \mathbf{K}_d) \mathbf{x}) - \ell_f(\mathbf{x}) + \mathbf{x}^T \mathbf{K}_d^T \mathbf{R}_d \mathbf{K}_d \mathbf{x} + \mathbf{x}^T \mathbf{Q}_d \mathbf{x} \leq 0, \quad \forall \mathbf{x} \in X_f. \quad (6)$$

The centralized MPC scheme for the leader-follower system described by dynamics (1)-(2) based on a terminal set-terminal cost approach, given an initial state \mathbf{x} and prediction horizon N , is formulated as follows:

$$V_N(\mathbf{x}) = \min_{\mathbf{x}, \mathbf{u}} \sum_{i=1}^M \sum_{t=0}^{N-1} \ell^i(x_t^i, u_t^i) + \ell_f(\mathbf{x}_N) \quad (7)$$

s.t: dynamics (1) and (2)

$$G_x^i x_t^i + G_u^i u_t^i \leq b^i, x_0^i = x^i, \forall i = 1, \dots, M, \\ \mathbf{x}_N \in X_f.$$

It is a well-known result [18] that the above MPC scheme, under assumptions A.1-A.3, stabilizes the system (4), with the optimal value of problem (7), $V_N(\mathbf{x})$, as a Lyapunov function. Keeping in line with the distributed nature of our system, the control law \mathbf{K}_d , the final stage cost ℓ_f and the terminal constraint set X_f need to be computed locally. In the following sections we develop a distributed synthesis procedure under such structural constraints.

A. Terminal Cost

For a locally computed \mathbf{K}_d , we employ distributed control laws $u^i = K^i x^i$ for each subsystem, with $K^i \in \mathbb{R}^{m_i \times n_i}$ and the resulting control law for the entire system will then be $\mathbf{u} = \mathbf{K}_d \mathbf{x}$, where the matrix $\mathbf{K}_d = \text{diag}(K^i)$ is block-diagonal. For the terminal stage cost, we define $\ell_f(\mathbf{x}) = \sum_{i=1}^M \ell_f^i(x^i)$, where terminal stage costs for each subsystem are of the following quadratic form:

$$\ell_f^i(x^i) = \frac{1}{2} \|x^i\|_{P^i}^2, \quad \forall i = 2, \dots, M,$$

where the matrix $P^i \in \mathbb{R}^{n_i \times n_i}$ is positive definite, such that $\mathbf{P}_d = \text{diag}(P^i)$.

Due to the block-diagonal structure of matrices \mathbf{P}_d , \mathbf{Q}_d and \mathbf{R}_d , we can rewrite (6) equivalently as the following inequality:

$$V_f^1(x^1) + \sum_{i=2}^M V_f^i(x^i, x^{i-1}) \leq 0, \quad \forall \mathbf{x} \in X_f,$$

¹In this paper, we use the following notation: $\|x\|_P^2 = x^T P x$.

where the left hand side is a sum of local functions V_f^i that have the following form:

$$V_f^1(x^1) = (x^1)^T \left((\tilde{A}^1)^T P^1 \tilde{A}^1 - P^1 + Q^1 + (K^1)^T R^1 K^1 \right) x^1$$

$$V_f^i(x^i, x^{i-1}) = [(x^i)^T \quad (x^{i-1})^T] \mathbf{P}^i \begin{bmatrix} x^i \\ x^{i-1} \end{bmatrix}, \quad \forall i \geq 2,$$

where $\tilde{A}^i = A^i + B^i K^i$, $\tilde{A}^{i,i-1} = A^{i,i-1} + B^{i,i-1} K^{i-1}$ and matrices \mathbf{P}^i are of the following form:

$$\begin{bmatrix} (\tilde{A}^i)^T P^i \tilde{A}^i - P^i + Q^i + (K^i)^T R^i K^i & (\tilde{A}^i)^T P^i \tilde{A}^{i,i-1} \\ (\tilde{A}^{i,i-1})^T P^i \tilde{A}^i & (\tilde{A}^{i,i-1})^T P^i \tilde{A}^{i,i-1} \end{bmatrix}.$$

We can ensure inequality (6) imposing the following distributed structure (see also [11] for a similar approach):

$$V_f^1(x^1) \leq q^1(x^1) \quad (8)$$

$$V_f^i(x^i, x^{i-1}) \leq q^i(x^i, x^{i-1}), \quad \forall i = 2, \dots, M, \quad \mathbf{x} \in X_f \quad (9)$$

such that:

$$q(\mathbf{x}) = q^1(x^1) + \sum_{i=2}^M q_i(x^i, x^{i-1}) \leq 0, \quad \forall \mathbf{x} \in X_f. \quad (10)$$

We consider that the functions q^i do not necessarily take negative values and have the following quadratic form:

$$q^1(x^1) = (x^1)^T W^1 x^1$$

$$q^i(x^i, x^{i-1}) = [(x^i)^T \quad (x^{i-1})^T] W^i \begin{bmatrix} x^i \\ x^{i-1} \end{bmatrix},$$

where the matrices $W^i = \begin{bmatrix} (W^i)_{11} & (W^i)_{12} \\ (W^i)_{12}^T & (W^i)_{22} \end{bmatrix}$ are symmetric. Clearly, $q(\mathbf{x})$ is also quadratic function and thus can be written as $q(\mathbf{x}) = \mathbf{x}^T W \mathbf{x}$, for an appropriate matrix W defined below. We now define the following optimization problem:

$$\begin{aligned} & \min_{P^i, K^i, W^i, \tau} \tau \\ & \text{s.t: } \mathbf{M}^i(P^i, K^i, W^i) \preceq 0, \quad \forall i = 1, \dots, M \\ & \quad W \preceq \tau I, \end{aligned} \quad (11)$$

where $\mathbf{M}^i(\cdot)$ refer to the matrix inequalities (8) and (9) and the matrix W has the following block tridiagonal structure:

$$\begin{bmatrix} W^1 + W_{22}^2 & W_{12}^2 & 0 & \dots & 0 \\ (W_{12}^2)^T & W_{11}^2 + W_{22}^3 & \dots & 0 & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & 0 & \dots & W_{11}^{M-1} + W_{22}^M & W_{12}^M \\ 0 & \dots & 0 & (W_{12}^M)^T & W_{11}^M \end{bmatrix}.$$

It is straightforward to see that if (11) has an optimal value $\tau^* \leq 0$, ensuring that $W \leq 0$ and subsequently (10) holds, then (6) is satisfied. Note that we do not require that matrices W_i to be negative semi-definite. On the contrary, positive or indefinite matrices allow local terminal costs to increase as long as the global cost still decrease. This approach reduces conservatism in deriving the matrices P_i and K_i . However, problem (11) is in a form that cannot be solved efficiently

since it is not a convex problem. Subsequently, we show that (11) can be expressed as a sparse SDP that can be solved distributively. To this goal, we employ the following linearization [22]:

$$P^i = (S^i)^{-1}, \quad K^i = Y^i G^{-1}. \quad (12)$$

We also define the following matrices in order to make constraints of the optimization problem in the following theorem more compact notationally:

$$\begin{aligned} \tilde{G}^1 &= G + G^T - S^1 + \tilde{W}^1 \\ \tilde{G}^i &= \begin{bmatrix} G + G^T - S^i + \tilde{W}_{11}^i & \tilde{W}_{12}^i \\ (\tilde{W}_{12}^i)^T & G + G^T - \mu^i I + \tilde{W}_{22}^i \end{bmatrix} \\ \tilde{B}^1 &= \begin{bmatrix} A^1 G + B^1 Y^1 \\ (Q^1)^{\frac{1}{2}} G \\ (R^1)^{\frac{1}{2}} Y^1 \end{bmatrix}, \\ \tilde{B}^i &= \begin{bmatrix} A^i G + B^i Y^i & A^{i,i-1} G + B^{i,i-1} Y^{i,i-1} \\ (Q^i)^{\frac{1}{2}} G & 0 \\ (R^i)^{\frac{1}{2}} Y^i & 0 \\ 0 & G^T \end{bmatrix} \\ \tilde{S}^1 &= \begin{bmatrix} S^1 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad \tilde{S}^i = \begin{bmatrix} S^i & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & \mu^i I \end{bmatrix}. \end{aligned}$$

Note that the linearizations (12) have been employed under the assumption that all the subsystems have the same dimension for the states, i.e. $n_i = n_j$ for all i, j .

Theorem 1: If the following SDP

$$\min_{G, S^i, Y^i, Y^{i,i-1}, \tilde{W}, \mu^i, \tau} \tau \quad (13)$$

$$\begin{aligned} \text{s.t: } & \begin{bmatrix} \tilde{G}^i & (\tilde{B}^i)^T \\ \tilde{B}^i & \tilde{S}^i \end{bmatrix} \succcurlyeq 0, \quad \forall i = 1, \dots, M \\ & Y^{i,i-1} = Y^{i-1}, \quad \forall i = 2, \dots, M \\ & \tilde{W} \preceq \tau I, \end{aligned} \quad (14)$$

where \tilde{W} has the same structure as W , has a negative optimal value $\tau^* < 0$, then (6) holds.

Proof: From (14) we observe that $S^i \succ 0$ and $\mu^i > 0$, which in turn implies:

$$\begin{aligned} & (S^i - G)^T (S^i)^{-1} (S^i - G) \succcurlyeq 0 \\ & (\mu^i I - G)^T \frac{1}{\mu^i} I (\mu^i I - G) \succcurlyeq 0. \end{aligned}$$

By adding \tilde{W}^i to the previous inequalities, we get:

$$\tilde{G}^i \preceq \begin{bmatrix} G^T & 0 \\ 0 & G^T \end{bmatrix} \begin{bmatrix} (S^i)^{-1} & 0 \\ 0 & \frac{1}{\mu^i} I \end{bmatrix} \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix} + \tilde{W}^i. \quad (15)$$

For $i = 2, \dots, M$, using (15) and the equality constraints $Y^{i,i-1} = Y^{i-1}$ and by applying the Schur complement to (14) we obtain:

$$\begin{aligned} \tilde{W}^i & \preceq \begin{bmatrix} (\tilde{A}^i)^T P^i \tilde{A}^i - P^i & (\tilde{A}^i)^T P^i \tilde{A}^{i,i-1} \\ (\tilde{A}^{i,i-1})^T P^i \tilde{A}^i & (\tilde{A}^{i,i-1})^T P^i \tilde{A}^{i,i-1} \end{bmatrix} \\ & + \begin{bmatrix} Q^i + (K^i)^T R^i K^i + G^{-T} G^{-1} & 0 \\ 0 & 0 \end{bmatrix}, \end{aligned}$$

which is equivalent to (9) if we take:

$$W^i = \begin{bmatrix} G^{-T} & 0 \\ 0 & G^{-T} \end{bmatrix} \begin{bmatrix} \tilde{W}_{11}^i & \tilde{W}_{12}^i \\ (\tilde{W}_{12}^i)^T & \tilde{W}_{22}^i \end{bmatrix} \begin{bmatrix} G^{-1} & 0 \\ 0 & G^{-1} \end{bmatrix}.$$

To transform inequality (8) into a linear matrix inequality of type (14), we use the same linearizations and the proof follows similar steps as those previously presented. As a result, the SDP (13) is equivalent to problem (11), and for a negative optimal value τ^* , (6) is satisfied. ■

Note that the SDP problem (13)-(14) can be solved offline either using a sparse SDP solver or some distributed optimization algorithm [23]. Since we have imposed a diagonal structure on the controller $\mathbf{K}_d = \text{diag}(K^i)$, it follows that the system matrix $\mathbf{A} + \mathbf{BK}_d$ has a block bidiagonal structure. If the optimal solution τ^* of the SDP is negative, then the matrix $\mathbf{A} + \mathbf{BK}_d$ is Schur (all the eigenvalues are strict inside the unit circle). It follows that all the matrices $A^i + B^i K^i$ are Schur.

B. Terminal Set

To complete the stability analysis for system (5), which implies properties A.1 - A.3, we need to complete the design procedure by the computation of a terminal set $X_f \subset \mathbb{R}^n$, defined locally (as a Cartesian product) $X_f = \prod_{i=1}^M X_f^i$ and equipped with invariance properties.

First let us define the set of *admissible states* associated to the constraints (3) and the specific linear controller K^i :

$$X^i = \{x^i : (G_x^i + G_u^i K^i)x^i \leq b^i\}.$$

leading via the Cartesian product to a set in \mathbb{R}^n :

$$X = \prod_{i=1}^M X^i.$$

Assumption 1: The origin is assumed to be an interior point of the set X .

We introduce the following formal definition of *positive invariance* in view of its use in the practical construction of the terminal set X_f .

Definiton 1: A set $\Omega \subseteq X$ is called positive invariant for system (5) if $\mathbf{x}_t \in \Omega$ it holds that $\mathbf{x}_{t+1} \in \Omega$ for all $t \geq 0$. As a standard approach in the MPC design [18], the terminal set $X_f \subset X$ needs to be positive invariant for the nominal linear time-invariant dynamics (5). This is a standard problem in set-theoretic control theory and there are a number of ways in which can be computed (see e.g [16], [6], [8]).

Due to the distributed nature of our system, such a general terminal constraint set cannot be used due to the introduction of coupling constraints between the states of the subsystems. We need to explore the possibility of finding a terminal constraint set, which preserves the structure of a Cartesian product:

$$X_f = \prod_{i=1}^M X_f^i, \quad (16)$$

This will further enable a distributed use of the terminal constraint sets X_f^i for each of the subsystems. Is worth mentioning that for general systems the construction of a

terminal set in the form given above can be cumbersome in distributed settings (see e.g. [5] for such a construction). However, for a system $\mathbf{x}_{t+1} = \tilde{\mathbf{A}}\mathbf{x}_t$, where $\tilde{\mathbf{A}}$ has a special block bidiagonal structure and the admissible set is expressed as $X = \prod_{i=1}^M X^i$, the computation of such an invariant set $X_f = \prod_{i=1}^M X_f^i$ can be simplified by exploiting these structural properties.

Without loss of generality the matrix $\tilde{\mathbf{A}}$ will be considered to be of the following form:

$$\tilde{\mathbf{A}} = \begin{bmatrix} \bar{A}^{11} & 0 & \dots & \dots & 0 \\ \bar{A}^{21} & \bar{A}^{22} & 0 & \dots & 0 \\ 0 & \bar{A}^{32} & \bar{A}^{33} & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \bar{A}^{M,M-1} & \bar{A}^{MM} \end{bmatrix}$$

i.e block lower-bidiagonal. The developments in Subsection II-A point to the construction of a distributed linear controller which allow us to assume the stability of the unconstrained local closed-loop system $\mathbf{x}_{t+1} = \tilde{\mathbf{A}}\mathbf{x}_t$ around the origin. By the block lower-bidiagonal structure it follows that the matrix $\tilde{\mathbf{A}}$ is Schur (i.e. $|\lambda(\tilde{\mathbf{A}})| < 1$) and consequently through the block lower-bidiagonal form of $\tilde{\mathbf{A}}$, all the matrices \bar{A}^{ii} are also Schur, for all $i = 1, \dots, M$.

The dynamics for the comprising subsystems are:

$$x_{t+1}^1 = \bar{A}^{11}x_t^1 \quad (17)$$

$$x_{t+1}^i = \bar{A}^{ii}x_t^i + \bar{A}^{i,i-1}x_t^{i-1}, \quad \forall i = 2, \dots, M. \quad (18)$$

1) *Construction of X_f^1 :* By taking into account that the first subsystem is stable and its dynamics are not perturbed by the other subsystems, the computation of $X_f^1 \subset X_1$ as a positive invariant set with respect to (17) can be done easily through standard methods for LTI nominal dynamics available in [6], [8].

We note also that the boundedness of the set X_1 will ensure boundedness properties for the set X_f^1 .

Remark 1: If $X_f^1 \subseteq X_1$ is invariant with respect to (17) and $0 \in \text{int}(X_f^1)$ then αX_f^1 is invariant and $0 \in \text{int}(\alpha X_f^1)$ for any scalar $\alpha > 0$. More than that, if $0 < \alpha < 1$, then $\alpha X_f^1 \subseteq X_1$.

2) *Completing the construction of X_f :* For the subsystems $i = 2, \dots, M$ we require a different treatment. If we denote $\bar{A}^{i,i-1}x_t^{i-1} = w_t^i$, the dynamics for the remaining subsystems can be considered as:

$$x_{t+1}^i = \bar{A}^{ii}x_t^i + w_t^i, \quad \forall i = 2, \dots, M, \quad (19)$$

where w_t^i can now be viewed as an unknown disturbance for this particular subsystem, where w_t^i is bounded, i.e w_t^i is in a set \mathcal{W}^1 . We denote by $w(\cdot) \in \mathcal{M}_{\mathcal{W}}$ the sequence w_0, w_1, \dots, w_k of disturbances from the admissible set $\mathcal{M}_{\mathcal{W}} = \{w(\cdot) | w_k \in \mathcal{W}, \forall k \in \mathbb{N}\}$.

¹In the case of the second subsystem $i = 2$, we have $w_t^2 = \bar{A}^{21}x_t^1$, and by taking into account that X_f^1 is positive invariant, it can be observed that w_t^2 is bounded, i.e $w_t^2 \in \mathcal{W}^2$, where $\mathcal{W}^2 = \bar{A}^{21}X_f^1$.

Definiton 2: The set $\mathcal{O} \subseteq X$ is a robust positive invariant set for a system $x_{t+1} = Ax_t + w_t$, if starting from \mathcal{O} , the evolution of the system remains in \mathcal{O} for all $w(\cdot) \in \mathcal{M}_W$.

We observe that X_f^i can now be computed as a robust positive invariant set (RPI) for the subsystem with the index $i \geq 2$, by exploiting the contractiveness properties of \bar{A}^{ii} and the existence of explicit bounds on w_t^i . The practical construction of such RPI sets is standard in the literature, see for example the procedures in [6], [14], [15]. In the following such a constructive procedure will be denoted by $X_f^i = RPI(X^i, \mathcal{W}^i)$.

Proposition 1: Let $X_f^i = RPI(X^i, \mathcal{W}^i)$ be an invariant set with respect to (18), having the origin as interior point. There always exists a scalar $0 < \alpha < 1$ such that $\alpha X_f^i = RPI(\alpha X^i, \alpha \mathcal{W}^i)$ preserve the invariance properties and additionally $\alpha X_f^i \subseteq X_i$.

Proof: The proof is an immediate application of the Remark 1 and the scaling properties of the RPI sets detailed in [13]. ■

With these (robust) positive invariance and constraint satisfaction properties we are able to propose a constructive procedure for X_f^i in a iterative manner, starting from the first subsystem and leading to an invariant set in \mathbb{R}^n , as presented in the following algorithm:

- 1) compute X_f^1
- 2) for $i = 2 : M$
 1. compute $\mathcal{W}^i = \bar{A}^{i,i-1} X_f^{i-1}$;
 2. compute $X_f^i = RPI(X^i, \mathcal{W}^i)$;
- 3) find $0 < \alpha < 1$ such that $\alpha X_f^i \subseteq X_i, \forall i = 1, \dots, M$

Since for the leader-follower systems described in this paper the matrix $\mathbf{A} + \mathbf{B}\mathbf{K}_d$ is block lower-bidiagonal as well, we can use the procedure described above to compute a terminal set of the form $X_f = \prod_{i=1}^M X_f^i$ that satisfies the properties A.1–A.3. Note that the distributed MPC controller presented below results in a larger region of attraction compared to other MPC schemes based on an end point constraint [2]. An additional novelty of our approach consists in the fact that all the computations for the terminal set and cost can be carried out in a completely distributed way. Note that this strategy for constructing sets X_f^i can also be extended to the case where $\tilde{\mathbf{A}}$ is block lower triangular, i.e subsystem $i \geq 2$ is affected by subsystems $1, \dots, i-1$. In this case, the sets \mathcal{W}^i would be constructed as $\mathcal{W}^i = \mathcal{W}^{i,1} \oplus \dots \oplus \mathcal{W}^{i,i-1}$, where by \oplus we denote the Minkowski sum: $A \oplus B = \{x + y | x \in A, y \in B\}$ and $\mathcal{W}^{i,j} = \bar{A}^{ij} X_f^j$, $j = 1, \dots, i-1$.

We can now reformulate the centralized MPC problem for the entire system (7) as following:

$$V_N(\mathbf{x}) = \min_{\mathbf{x}, \mathbf{u}} \sum_{i=1}^M \sum_{t=0}^{N-1} \ell^i(x_t^i, u_t^i) + \ell_f^i(x_N^i) \quad (20)$$

s.t: dynamics (1) and (2),

$$G_x^i x_t^i + G_u^i u_t^i \leq b^i, \quad G^i x_N^i \leq f^i, \quad \forall i = 1, \dots, M$$

where we assume that the terminal sets X_f^i constructed previously are polyhedra described by $G^i x_N^i \leq f^i$, with $f^i > 0$.

III. PROBLEM RESTRUCTURING

We now propose to reformulate problem (20) as to obtain a more suitable structure. We define the intermediary stage variables for subsystem i as:

$$\mathbf{x}_t^i = [(x_t^i)^T \quad (u_t^i)^T]^T \in \mathbb{R}^{\mathbf{n}_i},$$

where $\mathbf{n}_i = n_i + m_i$ and $t = 1, \dots, N-1$. Next, we define the general decision variable $\mathbf{z} \in \mathbb{R}^{\mathbf{n}}$ for (20) as follows:

$$\mathbf{z} = [(\mathbf{z}^1)^T \dots (\mathbf{z}^M)^T]^T,$$

where $\mathbf{n} = \sum_{i=1}^M N \mathbf{n}_i$ and

$$\mathbf{z}^i = [(u_0^i)^T \quad (x_1^i)^T \dots (u_{N-1}^i)^T \quad (x_{N-1}^i)^T \quad (x_N^i)^T]^T.$$

Now, in accordance with the general decision variable as defined above and in order to create a more compact and ordered final structure, we need to define the following matrices:

$$\begin{aligned} \mathbf{E}^i &= [I_{n_i} \quad 0] \in \mathbb{R}^{n_i \times \mathbf{n}_i}, \quad \mathbf{A}^i = [A^i \quad B^i] \in \mathbb{R}^{n_i \times \mathbf{n}_i} \\ \mathbf{A}^{i,i-1} &= [A^{i,i-1} \quad B^{i,i-1}] \in \mathbb{R}^{n_i \times \mathbf{n}_{i-1}} \\ \mathbf{Q}^i &= \text{diag}(Q^i, R^i) \in \mathbb{R}^{\mathbf{n}_i \times \mathbf{n}_i} \\ \tilde{\mathbf{Q}}^i &= \text{diag}(R^i, \mathbf{Q}^i, \dots, \mathbf{Q}^i, P^i), \end{aligned}$$

where $\tilde{\mathbf{Q}}^i \in \mathbb{R}^{N \mathbf{n}_i \times N \mathbf{n}_i}$ has $N-1$ \mathbf{Q}^i blocks in its diagonal. Using the intermediary stage variable we can rewrite the equality constraints in (20) for subsystem i as:

$$\mathbf{E}^i \mathbf{x}_{t+1}^i = \mathbf{A}^i \mathbf{x}_t^i + \mathbf{A}^{i,i-1} \mathbf{x}_t^{i-1}. \quad (21)$$

We now recast (20) as:

$$\begin{aligned} \min_{\mathbf{z}} \quad & \mathbf{z}^T H \mathbf{z} \\ \text{s.t.} \quad & G \mathbf{z} \leq b, \quad C \mathbf{z} = c, \end{aligned} \quad (22)$$

where $H \in \mathbb{R}^{\mathbf{n} \times \mathbf{n}}$ is $\text{diag}(\tilde{\mathbf{Q}}^i)$, with $i = 1, \dots, M$. We have included the equality constraints for each subsystem in (20) in $C \mathbf{z} = c$, where $c \in \mathbb{R}^{\sum_{i=1}^M N \mathbf{n}_i}$ and $C \in \mathbb{R}^{\sum_{i=1}^M N \mathbf{n}_i \times \mathbf{n}}$ are the following:

$$c = \begin{bmatrix} A^1 x_0^1 \\ 0_{(N-1)n_1,1} \\ A^{21} x_0^1 + A^2 x_0^2 \\ 0_{(N-1)n_2,1} \\ \vdots \\ A^{M,M-1} x_0^{M-1} + A^M x_0^M \\ 0_{(N-1)n_M,1} \end{bmatrix}$$

$$C = \begin{bmatrix} C^{11} & 0 & \dots & \dots & 0 \\ C^{21} & C^{22} & 0 & \dots & 0 \\ 0 & C^{32} & C^{33} & \dots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & \dots & 0 & C^{M,M-1} & C^{MM} \end{bmatrix}. \quad (23)$$

In (23) the matrices $C^{ii} \in \mathbb{R}^{Nn_i \times Nn_i}$, for $i = 1, \dots, M$, have the following structure:

$$C^{ii} = \begin{bmatrix} -B^i & \mathbf{E}^i & \dots & \dots & \dots & 0 \\ 0 & -\mathbf{A}^i & \mathbf{E}^i & \dots & \dots & 0 \\ 0 & 0 & -\mathbf{A}^i & \mathbf{E}^i & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & -\mathbf{A}^i & I_{n_i} \end{bmatrix},$$

whilst the matrices $C^{i,i-1} \in \mathbb{R}^{Nn_i \times Nn_{i-1}}$, for $i = 2, \dots, M$, have the following structure:

$$\begin{bmatrix} -B^{i,i-1} & 0 & \dots & \dots & \dots & 0 \\ 0 & -\mathbf{A}^{i,i-1} & 0 & \dots & \dots & 0 \\ \vdots & \vdots & -\mathbf{A}^{i,i-1} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & -\mathbf{A}^{i,i-1} & 0 \end{bmatrix}.$$

The inequality constraints in (20) have been recast as $G\mathbf{z} \leq b$, where $b \in \mathbb{R}^q$, with $\mathbf{q} = \sum_{i=1}^M (Nq_i + q)$ and $G \in \mathbb{R}^{q \times n}$ have the following structure:

$$b = [(\mathbf{b}^1)^T, \dots, (\mathbf{b}^M)^T]^T,$$

where

$$\mathbf{b}^i = [(b^i - G_x^i x_0^i)^T, \overbrace{(b^i)^T \dots (b^i)^T}^{N-1 \text{ times}}, (f^i)^T]^T,$$

and

$$G = \begin{bmatrix} \mathbf{G}^1 & 0 & \dots & 0 \\ 0 & \mathbf{G}^2 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \mathbf{G}^M \end{bmatrix}, \quad (24)$$

whose matrix blocks $\mathbf{G}^i \in \mathbb{R}^{Nq_i + q \times Nn_i}$ are:

$$\mathbf{G}^i = \begin{bmatrix} G_u^i & 0 & \dots & \dots & \dots & 0 \\ 0 & G_x^i & G_u^i & 0 & \dots & 0 \\ \vdots & 0 & 0 & \ddots & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & G_x^i & G_u^i \\ 0 & 0 & 0 & 0 & 0 & G^i \end{bmatrix}$$

IV. PRIMAL-DUAL INTERIOR POINT METHOD

Primal-dual interior point methods are very efficient optimization methods which employ the Karush-Kuhn-Tucker (KKT) conditions, that are both necessary and sufficient for achieving optimality for a convex optimization problem. We intend to use a primal-dual interior point algorithm for problem (22), which uses Mehrotra's predictor-corrector scheme [21]. The KKT optimality conditions which result from (22) are:

$$\begin{aligned} H\mathbf{z} + C^T \nu + G^T \lambda &= 0 \\ C\mathbf{z} - c &= 0 \\ G\mathbf{z} - b + s &= 0 \\ \Lambda S &= 0 \\ \lambda \geq 0, s \geq 0, \end{aligned}$$

where $s \in \mathbb{R}^q$ are slack variables, $\nu \in \mathbb{R}^{n_M}$ and $\lambda \in \mathbb{R}^q$ are the Lagrange multipliers and $S = \text{diag}(s)$, $\Lambda = \text{diag}(\lambda)$ are diagonal matrices formed from the slack variables and respective multipliers. These conditions lead to the following Newton system (see [1] for more details):

$$\begin{bmatrix} H & C^T & G^T & 0 \\ C & 0 & 0 & 0 \\ G & 0 & 0 & I \\ 0 & 0 & S & \Lambda \end{bmatrix} \begin{bmatrix} \Delta z \\ \Delta \nu \\ \Delta \lambda \\ \Delta s \end{bmatrix} = - \begin{bmatrix} r_z \\ r_\nu \\ r_\lambda \\ r_s \end{bmatrix}. \quad (25)$$

We can eliminate Δs by using $\Delta s = -\Lambda^{-1}(r_s + S\Delta\lambda)$. Furthermore, by reducing $\Delta\lambda = S^{-1}\Lambda(r_\lambda + G\Delta z) - S^{-1}r_s$, we obtain the following system:

$$\begin{bmatrix} \Phi & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \Delta z \\ \Delta \nu \end{bmatrix} = - \begin{bmatrix} r_d \\ r_\nu \end{bmatrix}, \quad (26)$$

where

$$\begin{aligned} \Phi &= H + G^T S^{-1} \Lambda G \\ r_d &= r_z + G^T S^{-1} \Lambda r_\lambda - G^T S^{-1} r_s. \end{aligned} \quad (27)$$

Next, we form the Schur complement of the matrix in (26) so as to obtain the final system of equations:

$$Y\Delta\nu = \tau \quad (28)$$

$$Y = C\Phi^{-1}C^T \quad (29)$$

$$\tau = -r_\nu - C\Phi^{-1}r_d \quad (30)$$

$$\Delta z = \Phi^{-1}(-r_d - C^T \Delta \nu). \quad (31)$$

Solving (28) would normally employ the computation of Y , which may appear to be overwhelming given the large dimensions of Y and the fact that it requires an inversion of Φ . However, due to the way in which matrix Y is formed in (29), we show that we can compute its Cholesky factorization in an efficient and distributed manner, similar to the one found in [3] for one linear system. The matrix $\Phi \in \mathbb{R}^{n \times n}$ has a block-diagonal structure $\Phi = \text{diag}(\Phi^i)$, where the blocks $\Phi^i \in \mathbb{R}^{Nn_i \times Nn_i}$ are also block diagonal, with their first block of size $m_i \times m_i$, the following $N-1$ blocks of size $n_i \times n_i$ and the final block of size $n_i \times n_i$. Now, it can be observed that resulting matrix Y has the following block-tridiagonal structure:

$$\begin{bmatrix} Y^{11} & (Y^{21})^T & 0 & \dots & 0 & 0 \\ Y^{21} & Y^{22} & (Y^{32})^T & \dots & 0 & 0 \\ 0 & Y^{32} & Y^{33} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & Y_{M-1M-1} & Y_{MM-1}^T \\ 0 & 0 & 0 & \dots & Y_{MM-1} & Y_{MM} \end{bmatrix}$$

where the matrix blocks are:

$$\begin{aligned} Y^{11} &= C^{11}(\Phi^1)^{-1}(C^{11})^T \\ Y^{ii} &= C^{i,i-1}(\Phi^{i-1})^{-1}(C^{i,i-1})^T \\ &\quad + C^{ii}(\Phi^i)^{-1}(C^{ii})^T, \forall i = 2 \dots M \\ Y^{i,i-1} &= C^{i,i-1}(\Phi^{i-1})^{-1}(C^{i-1,i-1})^T, \forall i = 2 \dots M. \end{aligned} \quad (32)$$

First, we show how to compute efficiently in a distributed fashion the matrix Y . Note that inverting the block components of Φ and then forming the block components of Y would be very inefficient. However, if we form the Cholesky factorization of $\Phi^i = \mathbf{L}^i(\mathbf{L}^i)^T$ we get:

$$V^{ii} = C^{ii}(\mathbf{L}^i)^{-T} \quad (33)$$

$$W^{i,i-1} = C^{i,i-1}(\mathbf{L}^{i-1})^{-T}, \quad (34)$$

where $\mathbf{L}^i \in \mathbb{R}^{Nn_i \times Nn_i}$ are also block diagonal, so that the block components of Y are:

$$\begin{aligned} Y^{11} &= V^{11}(V^{11})^T \\ Y^{i,i} &= W^{i,i-1}(W^{i,i-1})^T + V^{ii}(V^{ii})^T, \quad \forall i \geq 2 \\ Y^{i,i-1} &= W^{i,i-1}(V^{i-1,i-1})^T, \quad \forall i \geq 2 \end{aligned}$$

The most efficient computation of $V^{i,i-1}$ can be done by solving the following systems of matrix equations, where the matrices L_j^i , with $j = 0, \dots, N$, are the diagonal elements of \mathbf{L}^i :

$$L_0^i(V_{11}^i)^T = (B^i)^T \quad (35)$$

$$L_j^i(V_{jj}^i)^T = (\mathbf{A}^i)^T, \quad \forall j = 1 \dots N-1 \quad (36)$$

$$L_j^i(V_{j,j+1}^i)^T = (\mathbf{E}^i)^T, \quad \forall j = 1 \dots N-1 \quad (37)$$

$$L_N^i(V_{N,N+1}^i)^T = I_{n_i}. \quad (38)$$

Equations (35)–(38) can be efficiently solved by matrix forward substitution, due to the lower triangular form of L_j^i . The resulting matrix will take the following form:

$$V^{ii} = \begin{bmatrix} V_{11}^i & V_{12}^i & 0 & \dots & \dots & 0 \\ 0 & V_{22}^i & V_{23}^i & 0 & \dots & 0 \\ \vdots & 0 & V_{33}^i & V_{34}^i & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & V_{N,N}^i & V_{N,N+1}^i \end{bmatrix}.$$

To obtain $W^{i,i-1}$ we solve the following series of matrix equations, by matrix forward substitution, considering the dense nature of $B^{i,i-1}$ and $\mathbf{A}^{i,i-1}$:

$$L_0^{i-1}(W_{11}^i)^T = (B^{i,i-1})^T \quad (39)$$

$$L_j^{i-1}(W_{j+1,j+1}^i)^T = (\mathbf{A}^{i,i-1})^T, \quad \forall j = 1 \dots N-1, \quad (40)$$

where L_j^{i-1} are the diagonal elements of \mathbf{L}^{i-1} . The resulting $W^{i,i-1}$ matrices will have a block-diagonal structure.

Second, the resulting structure of the Cholesky factorization of $Y = LL^T$ is as follows:

$$L = \begin{bmatrix} L^{11} & 0 & 0 & \dots & 0 & 0 \\ L^{21} & L^{22} & 0 & \dots & 0 & 0 \\ 0 & L^{32} & L^{33} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & L^{M-1,M-1} & 0 \\ 0 & 0 & 0 & \dots & L^{M,M-1} & L^{MM} \end{bmatrix}$$

where $L^{ii} \in \mathbb{R}^{Nn_i \times Nn_i}$, $\forall i = 1, \dots, M$ and $L^{i,i-1} \in \mathbb{R}^{Nn_i \times Nn_{i-1}}$, $\forall i = 2 \dots M$. The block components L^{ii} and

$L^{i,i-1}$ can be obtained from the following:

$$L^{11}(L^{11})^T = Y^{11} \quad (41)$$

$$L^{i,i-1}(L^{i-1,i-1})^T = Y^{i,i-1} \quad (42)$$

$$L^{ii}(L^{ii})^T = Y^{ii} - L^{i,i-1}(L^{i,i-1})^T, \quad \forall i \geq 2. \quad (43)$$

Note that the matrices Y^{ii} have a block tridiagonal structure, but $L^{i,i-1}$ are usually dense so that there is no special structure in the terms $Y^{ii} - L^{i,i-1}(L^{i,i-1})^T$. Therefore, the Cholesky factorization of these matrices is computationally demanding.

V. DISCUSSION ON IMPLEMENTATION

The most important aspect of the algorithm previously presented is that it can be implemented in a distributed manner, between the M subsystems. The Cholesky factorization of Y clearly dominates the system of equations (28) when it comes to computing cost. By computing the matrices $V^{i,i}$ and $W^{i,i-1}$, the inversion of Φ can be avoided, and they can further be used in (30) and (31) to calculate the respective residuals. The factorization of Y is also the most complex when it comes to the communication between subsystems, requiring the back and forth transmission of matrices between subsystems. Also, due to the structure of Y , the factorization cannot be done in parallel and is achieved in a sequential manner. For subsystem i , with $i = 2, \dots, M-1$ the following steps are required for obtaining L^{ii} and $L^{i,i-1}$:

- 1) Compute $\Phi_i = \mathbf{L}_i \mathbf{L}_i^T$
- 2) Send \mathbf{L}^i to subsystem $i+1$
- 3) Receive \mathbf{L}^{i-1} from subsystem $i-1$
- 4) Compute V^{ii} : solve (35) to (38)
- 5) Send V^{ii} to subsystem $i+1$
- 6) Compute $W^{i,i-1}$: solve (39) and (40)
- 7) Compute Y^{ii} , receive $V^{i-1,i-1}$ from subsystem $i-1$
- 8) Compute $Y^{i,i-1}$, receive $L^{i-1,i-1}$ from subsystem $i-1$
- 9) Compute $L^{i,i-1}$ from (42)
- 10) Compute L^{ii} from (43), send $L^{i,i}$ to subsystem i

The number of flops for computing the Cholesky factorization of Y by each subsystem are provided in Table I:

TABLE I
NUMBER OF FLOPS FOR COMPUTING LOCAL COMPONENTS OF THE CHOLESKY FACTORIZATION OF Y

Operation	Number of flops (approximate)
Factor: $\Phi^i = \mathbf{L}^i(\mathbf{L}^i)^T$	$(N-1)\frac{n_i^3}{3} + \frac{n_i^3+m_i^3}{3}$
Solve: (35)	$n_i m_i^2$
Solve: (36), (37)	$2(N-1)n_i n_i^2$
Solve (38)	$\frac{n_i^3}{3}$
Solve (39)	$n_i m_{i-1}^2$
Solve (40)	$(N-1)n_i n_{i-1}^2$
Compute: Y^{ii}	$N n_i^2(n_i + n_i + n_{i-1} + 2)$
Compute: $Y^{i,i-1}$	$N(n_i n_{i-1} n_{i-1})$
Compute: L^{ii}	$\frac{N^3 n_i^3}{3}$
Compute: $L^{i,i-1}$	$N^3 n_{i-1}^2 n_i$

It can be observed that the matrices transmitted back and forth are very sparse, with a known block structure such that the only data required to be transmitted are these comprising blocks. Also, these blocks are transmitted only to neighboring subsystems, such that the transmission of data is localized.

Note that the cost of computing matrices L^{ii} and $L^{i,i-1}$ is cubic in N but linear in M overall, given the choice of \mathbf{z} . Also, computations can be done sequentially and exchange of information is only between neighbors. If we would rearrange \mathbf{z} by the prediction horizon, instead of by subsystems, then the dominating cost for computing these matrices would be linear in N overall and cubic in M locally, i.e. of order $\frac{(\sum_{i=1}^M n_i)^3}{3}$ for L^{ii} . However, this would imply that every subsystem has knowledge of the dynamics of all other subsystems, and as a result computations would require all-to-all transmission of data between subsystems. Thus, the efficient choice of \mathbf{z} given a physical leader-follower system involves the imposed prediction horizon N , the number of the subsystems M and possible transmission limitations between subsystems.

VI. CONCLUSIONS

In this paper we have showed that by restructuring certain MPC problems for large-scale systems we can reduce the computational cost of implementing an interior point algorithm for solving such problems. An analysis for obtaining a stabilizing linear control law from a distributed viewpoint has been made. By combining several recent results, we have proved that the online computation of MPC control laws for some special classes of large scale systems can be carried out with increased speed through a reduction of the number of required flops. This, in combination with ever-increasing distributed computing power that can be used for distributed computation of an MPC law suggests us that MPC can be used now in many large-scale applications where it has not been considered applicable before.

Further details regarding the efficient transmission of data between subsystems and the implementation results for the interior point method presented are omitted for lack of space.

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